René Schilling

On the domain of fractional Laplacians and related generators of Feller processes

Joint work with F. Kühn (TU Dresden)

We study the domain of the generator of stable processes, stable-like processes and more general pseudo- and integro-differential operators which naturally arise both in analysis and as infinitesimal generators of Lévy- and Lévy-type (Feller) operators. In particular we obtain conditions on the symbol of the operator ensuring that certain (variable order) Hölder and Hölder–Zygmund spaces are in the domain. We use tools from probability theory to investigate the smalltime asymptotics of the generalized moments of a Lévy or Lévy-type process $(X_t)_{t\geq 0}$,

$$\lim_{t \to 0} \frac{1}{t} \left(\mathbb{E}^x f(X_t) - f(x) \right), \quad x \in \mathbb{R}^d,$$

for functions f which are not necessarily bounded or differentiable. The pointwise limit exists for fixed $x \in \mathbb{R}^d$ if f satisfies a Hölder condition at x. Moreover, we give sufficient conditions which ensure that the limit exists uniformly in the space of continuous functions vanishing at infinity. As an application we prove that the domain of the generator of $(X_t)_{t\geq 0}$ contains certain Hölder spaces of variable order. Our results apply, in particular, to stable-like processes, relativistic stable-like processes, solutions of Lévy-driven SDEs and Lévy processes.