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General limit theory for Lévy driven moving average processes

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Lévy driven moving average (LDMA) processes are a class of null-spatial ambit fields, defined as

$$X_t := \int_{-\infty}^{t} g(t-s) - g_0(-s)\, dL_s,$$

where L is a Lévy process and g and g_0 are deterministic functions. This class of stochastic processes contains in particular the linear fractional stable motions, which are a natural generalisation of fractional Brownian motion. For LDMA processes driven by a pure jump Lévy process, we investigate the limit theory of general variation functionals of the form

$$V(f)_t = \sum_{i=1}^{[tn]} f(a_n \Delta_i^n X),$$

where f is an arbitrary continuous function, (a_n) a suitable deterministic normalising sequence, and $\Delta_i^n X = X_{i/n} - X_{(i-1)/n}$. The limiting behaviour of $V(f)_t$ depends on the Blumenthal-Getoor index of the driving Lévy process as well as on the shape of the kernel function g at 0. We derive several first order and second order limit theorems, generalising existing results on the power variation.