# Large Deviations for the Rough Bergomi Model

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CREATES Center for Research in Econometric Analysis of Time Series

Implied volatility modelling

Intermezzo: an introduction to large deviations

LDP for the rough Bergomi model

# Review of Black–Scholes option pricing

## The Black–Scholes model

In the Black–Scholes (1973) model, under the unique pricing measure $\mathbf{Q}$, the price of the underlying follows

$$dS_t = \sigma S_t dB_t,$$

where $\sigma > 0$ is the volatility parameter and $B$ is a standard Brownian motion under $\mathbf{Q}$.

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Consider a call option struck at $K = S_0 e^k > 0$ (that is, at log strike $k \in \mathbb{R}$) at time 0, paying

$$(S_T - K)^+ = (S_T - S_0 e^k)^+$$

units of cash at expiry $T > 0$.

# Review of Black–Scholes option pricing (cont'd)

## The Black–Scholes pricing formula

The unique arbitrage-free price of this call option under interest rate $r \geq 0$ is

$$C_{BS}(k, T; \sigma) = \mathbf{E}^{\mathbf{Q}}[e^{-rT}(S_T - S_0 e^k)^+] = S_0(\Phi(d_1) - \Phi(d_2)e^{k-rT}),$$

where

$$d_1 := \frac{1}{\sigma\sqrt{T}}\left(\left(r + \frac{\sigma^2}{2}\right)T - k\right),$$
$$d_2 := d_1 - \sigma\sqrt{T},$$

and $\Phi$ is the standard normal CDF.

# Implied volatility

## Black–Scholes implied volatility

The function $\sigma \mapsto C_{BS}(k, T; \sigma)$ is increasing.

So given a market quote $\widehat{C}(k, T)$, we can find $\hat{\sigma}$ such that

$$C_{BS}(k, T; \hat{\sigma}) = \widehat{C}(k, T).$$

The solution $\hat{\sigma} = \hat{\sigma}(k, T)$ is the (Black–Scholes) implied volatility of the quote $\widehat{C}(k, T)$.

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In practice, option traders prefer to quote prices in implied volatilities.

But it does not mean they believe in the Black–Scholes model!

# Implied volatility smile

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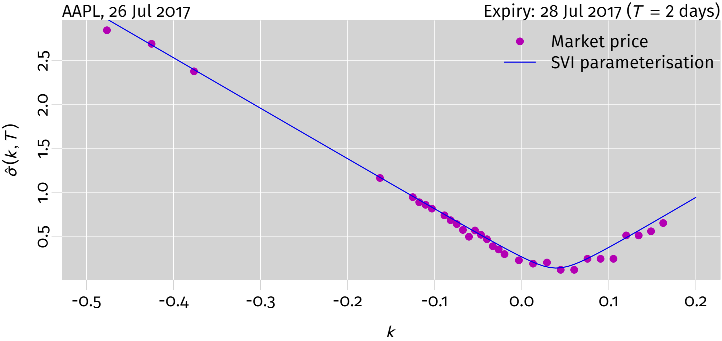
Instead of a flat line, the graph of $k \mapsto \hat{\sigma}(k, T)$ is U-shaped, depicting a smile.



# Reproducing the smile and skew

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## At-the-money skew

However, conventional stochastic volatility models, like the Heston (1993) model, are unable to reproduce the term structure of the at-the-money (ATM) skew

$$\psi(T) = \left| \frac{\partial}{\partial k} \hat{\sigma}(k, T) \right|_{k=0},$$

which in equity markets typically behaves near expiry as

$$\psi(T) \sim \text{const} \cdot T^{\alpha}, \quad T \to 0,$$

for some $\alpha$ slightly above $-\frac{1}{2}$.

# Rough Bergomi model

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## Rough Bergomi model

In the rough Bergomi model, under a pricing measure **Q**,

$$dS_t = \sqrt{v_t} S_t dB_t,$$

where

$$v_t := v_0 \exp\left(Z_t - \frac{\eta^2}{2} t^{2\alpha+1}\right), \quad Z_t := \eta\sqrt{2\alpha+1}\int_0^t (t-s)^\alpha dW_s$$

$S_0, v_0, \eta > 0$, $\alpha \in (-\frac{1}{2}, 0)$, and $B$ and $W$ are standard Brownian motions with $\langle B, W\rangle_t = \rho t$ for some $\rho \in (-1, 1)$.

# Rough Bergomi model (cont'd)

The instantaneous variance process $v$ is driven by the (rough) Riemann–Liouville process

$$\int_0^t (t-s)^\alpha dW_s, \quad t \geq 0,$$

whose sample paths are locally $\alpha + \frac{1}{2} - \varepsilon$-Hölder continuous.

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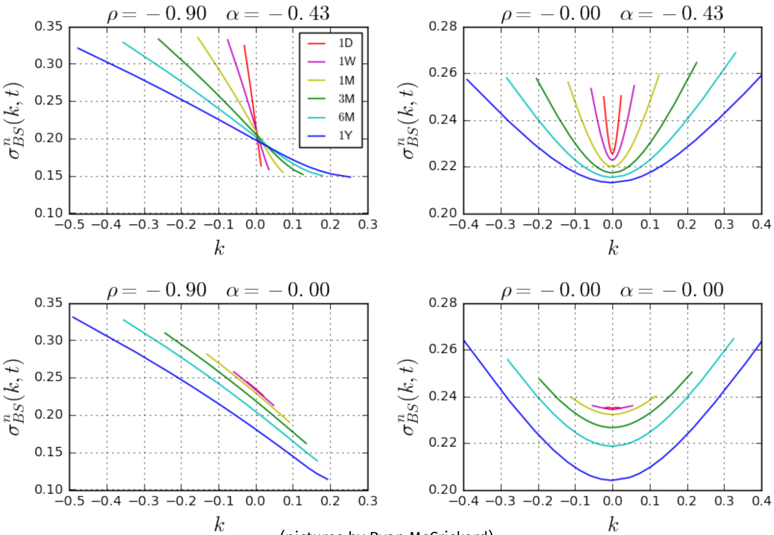
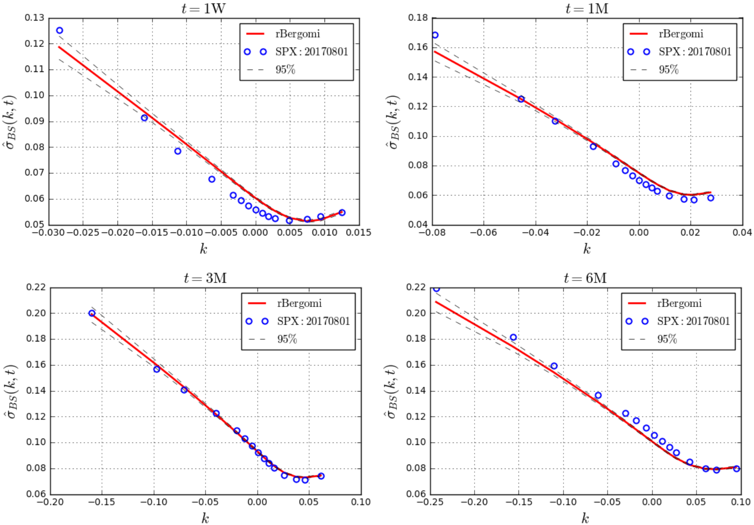
Even call and put options need to be priced by Monte Carlo — although efficient methods are available (Bennedsen, Lunde, and P., 2017$^{+}$; McCrickerd and P., 2017).

# Example: Rough Bergomi smiles




(pictures by Ryan McCrickerd)

# Example: Rough Bergomi calibration




$\eta = 2.54, \rho = -0.99, \alpha = -0.43$ (pictures by Ryan McCrickerd)

Implied volatility modelling

## Intermezzo: an introduction to large deviations

LDP for the rough Bergomi model

# Quantifying probabilities of rare events

Let $Y_1, \ldots, Y_n$ be iid random variables such that $|Y_1| \leq 1$ and $\mathbf{E}(Y_1) = 0$. Moreover, let $M_n$ be the sample mean of $Y_1, \ldots, Y_n$.

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Hoeffding's inequality says that, in fact, for all $n \in \mathbb{N}$ and $y > 0$,

$$\mathbf{P}[M_n \geq y] \leq \exp(-ny^2/2).$$

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Large deviations theory aims to give sharp exponential estimates of such probabilities.

# Large deviations principle

## Definition

A sequence $(X_n)_{n=1}^{\infty}$ of random elements in a Polish space $\mathbb{X}$ satisfies the large deviations principle (LDP) as $n \to \infty$ with speed $a_n \to \infty$ and rate function $I : \mathbb{X} \to [0, \infty]$ if

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## Remark

We can also consider a family $(X_\varepsilon)_{\varepsilon>0}$ of random elements and define the LDP as $\varepsilon \to 0$ analogously.

# Example: Cramér's theorem

Let $Y_1, \ldots, Y_n$ be iid rvs in $\mathbb{X} = \mathbb{R}$ and $M_n$ their sample mean.
Write $\psi(\theta) := \log \mathbf{E}[\exp(\theta Y_1)] \in (0, \infty]$ for $\theta \in \mathbb{R}$.

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## Theorem (Cramér, 1938; Varadhan, 1966)

*The sequence $(M_n)_{n=1}^{\infty}$ satisfies the LDP as $n \to \infty$ with speed $n$ and rate function*

$$I(x) = \sup_{\theta \in \mathbb{R}} \left(\theta x - \psi(\theta)\right), \quad x \in \mathbb{R},$$

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In particular, if $\mathbf{E}[|Y_1|] < \infty$ and $\mathbf{E}[Y_1] = 0$, then

$$\lim_{n\to\infty} \frac{1}{n} \log \mathbf{P}[M_n \geq y] \leq -I(y), \quad y > 0.$$

# Example: Schilder's theorem

Let $\varepsilon > 0$. Define a random element $X^\varepsilon$ of $\mathbb{X} = C([0,1])$ by

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where $W$ is a standard Brownian motion. Then $X^\varepsilon \xrightarrow{\mathbf{P}} 0$ in $(C([0,1]), \|\cdot\|_\infty)$ as $\varepsilon \to 0$.

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## Theorem (Schilder, 1966)

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$$I(x) = \begin{cases} \frac{1}{2}\int_0^1 x'(t)^2 dt & \text{if } x \in C([0,1]) \text{ is absolutely continuous,} \\ \infty & \text{otherwise.} \end{cases}$$

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This is an example of a functional LDP.

Implied volatility modelling

Intermezzo: an introduction to large deviations

## LDP for the rough Bergomi model

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- Which is why we make a detour and derive first a functional LDP (à la Schilder) for a rescaled version of $X$.
- Related results have been recently obtained by Bayer, Friz, Gulisashvili, Horvath, and Stemper (2017).

# Rescaled rough Bergomi model

## Rescaling

We define the rescaled version of the rough Bergomi log price $X_t = \log(S_t/S_0)$ by

$$X_t^\varepsilon := \int_0^t \sqrt{v_s^\varepsilon} dB_s^\varepsilon - \frac{1}{2}\int_0^t v_s^\varepsilon ds, \qquad B_t^\varepsilon := \varepsilon^{\beta/2} B_t,$$

$$v_t^\varepsilon := \varepsilon^{1+\beta} v_0 \exp\left(Z_t^\varepsilon - \frac{\eta^2}{2}(\varepsilon t)^\beta\right), \qquad Z_t^\varepsilon := \varepsilon^{\beta/2} Z_t,$$

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for any $t \in [0,1]$ and $\varepsilon > 0$, where $\beta = 2\alpha + 1 \in (0,1)$.

The rescaled process satisfies $X_\varepsilon \overset{d}{=} X_1^\varepsilon$ for any $\varepsilon > 0$.

# Functional LDP for the rough Bergomi model

## Theorem (Jacquier, P., and Stone, 2017)

*The family $(X^\varepsilon)_{\varepsilon>0}$ satisfies the LDP as $\varepsilon \to 0$ with speed $\varepsilon^{-\beta}$ and rate function*

$$I(x) = \begin{cases} \inf\left\{\frac{1}{2}\int_0^1 f(t)^2 dt : f \in L^2([0,1]), x = \mathcal{I}(f)\right\}, & x \in Ran(\mathcal{I}), \\ \infty, & x \notin Ran(\mathcal{I}), \end{cases}$$

*where* $\mathcal{I} : L^2([0,1]) \to C([0,1])$ *is some (quite complicated) non-linear integral operator.*

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*where $\mathcal{I} : L^2([0,1]) \to C([0,1])$ is some (quite complicated) non-linear integral operator.*

The proof is largely based on a generalised Schilder's theorem (Deuschel and Stroock, 1989), the contraction principle for LDPs, and the LDP for stochastic integrals by Garcia (2008).

# Univariate LDP and implied volatility asymptotics

Since $X_\varepsilon \overset{d}{=} X_1^\varepsilon$, we get by the contraction principle:

## Corollary

*The family $(X_\varepsilon)_{\varepsilon>0}$ (in $\mathbb{R}$) satisfies the LDP as $\varepsilon \to 0$ with speed $\varepsilon^{-\beta}$ and rate function $I_1(x) = \inf\{I(f) : f(1) = x\}$.*

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The methodology of Jacquier and Forde (1999) implies then:

## Corollary

*Under the rough Bergomi model, for $x \neq 0$,*

$$\lim_{T\to 0} T^{1+\beta} \hat{\sigma}(xT^{-\beta}, T)^2 = \begin{cases} \frac{x^2}{2\inf_{y\geq x} I_1(y)}, & x > 0, \\ \frac{x^2}{2\inf_{y\leq x} I_1(y)}, & x < 0. \end{cases}$$

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