Corporate Technology

Multifractal Design of Wind Fields

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Improve Wind-Field Modelling to Increase the Engineering Integrity of Wind Turbines

- External wind conditions affect structural loading, durability, and operation of wind turbines. Efficient grid control requires forecast of power production and fluctuation.
- Current standard (IEC 61400) for normal turbulence modeling of wind fields is based on Gaussian statistics.
- Observed turbulent wind fields are highly non-Gaussian.

Turbulence = Energy Cascade

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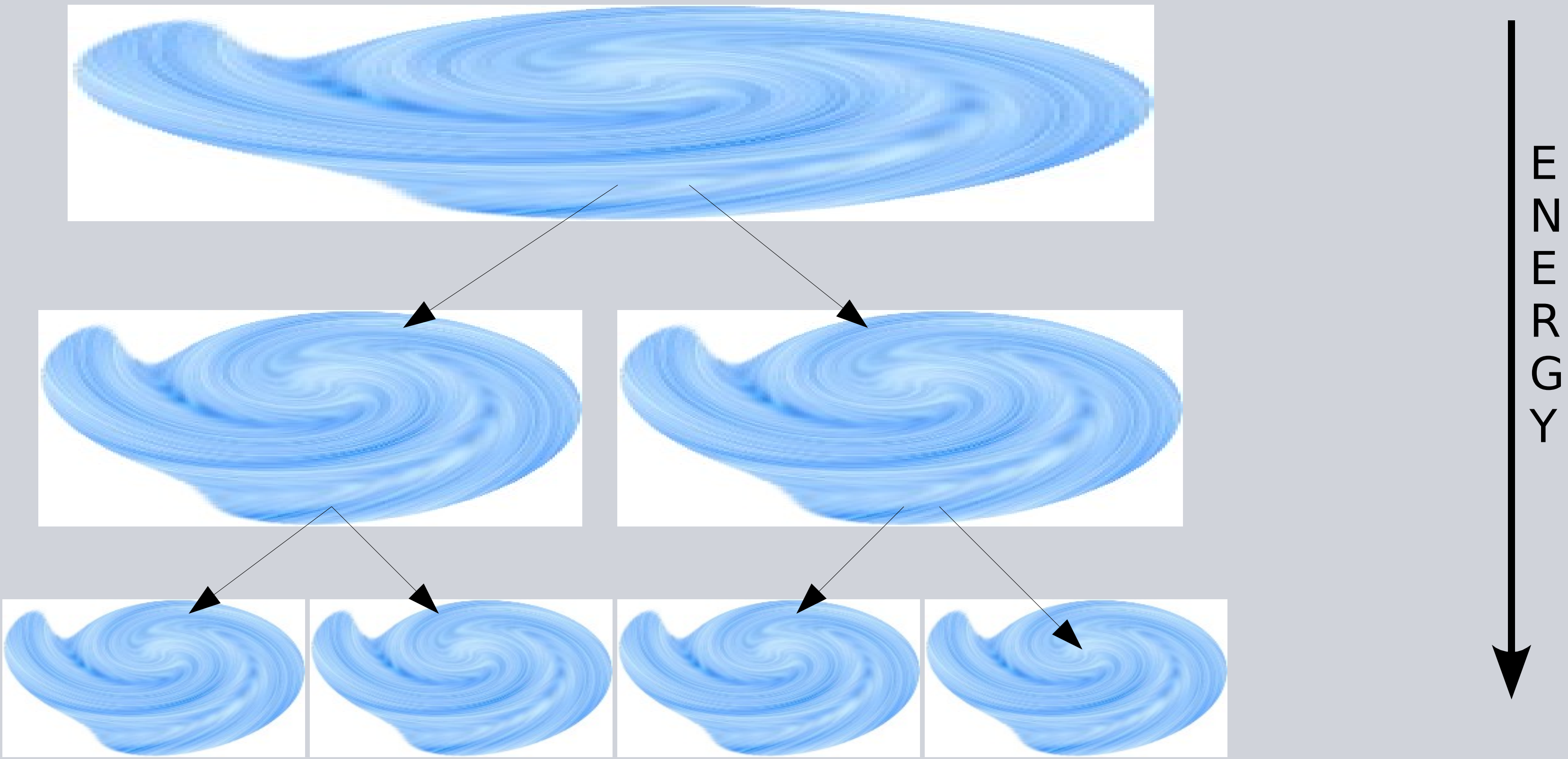

Classic Scaling

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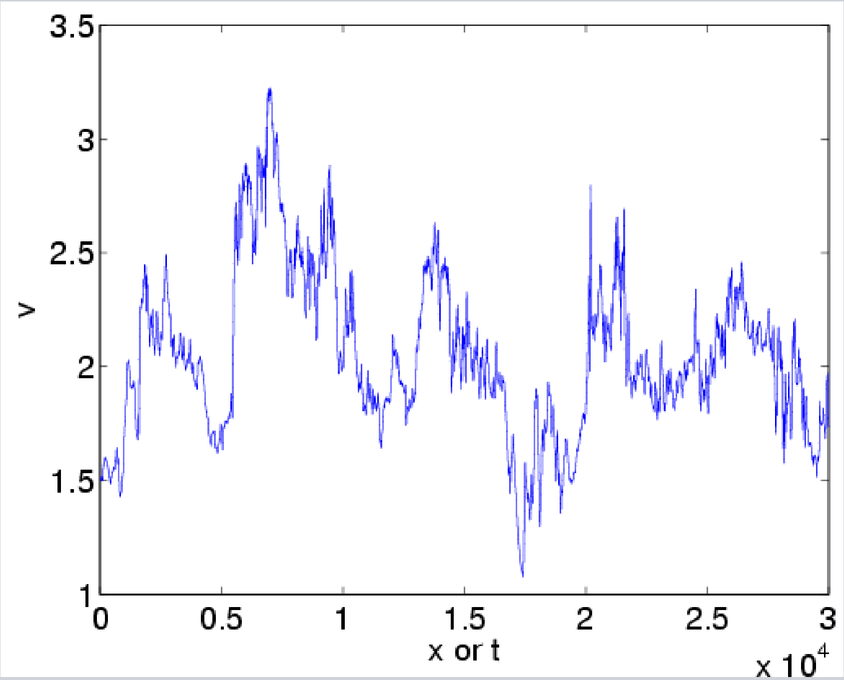

atmospheric boundary layer **R_λ = 9000**

Kolmogorov phenomenology:

$$\langle \Delta v_l^n \rangle \propto l^{\zeta_n}$$

$$\langle \Delta v_l^n \rangle \propto \langle \varepsilon_l^{n/3} \rangle \, l^{n/3}$$

$$\langle \varepsilon_l \rangle \propto l^{\tau_n}$$

$$(\eta << l << L)$$

$$\Delta v_l = [v(r+l) - v(r)] \cdot \hat{l}$$

$$\varepsilon_l = \frac{1}{l} \int_{x-l/2}^{x+l/2} \varepsilon(x')\, dx'$$

Scaling in Practice not Satisfactory

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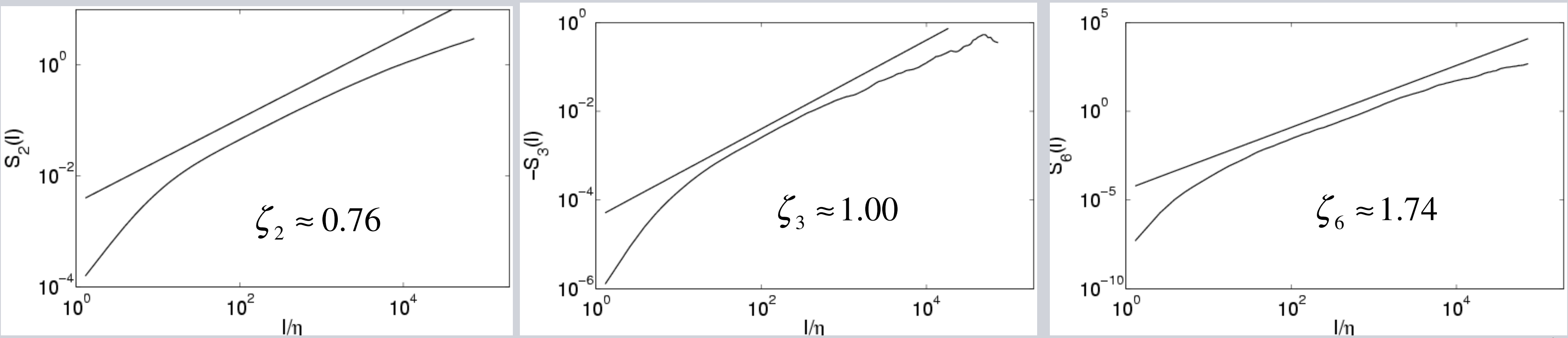
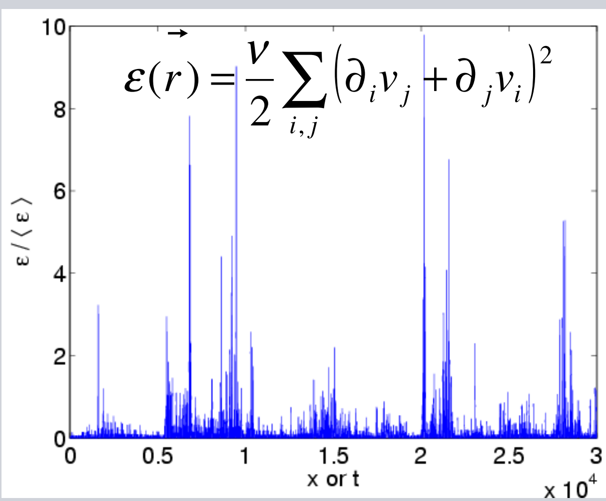
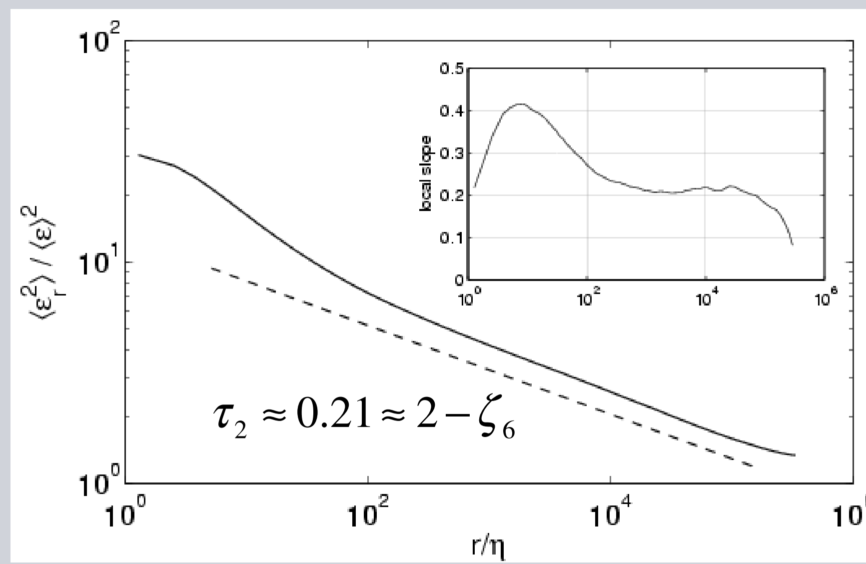


$\eta << l << L \qquad (L/\eta \approx 5 \cdot 10^4)$





$\eta << l \leq L$

Good Scaling of Energy Dissipation n-point Correlation

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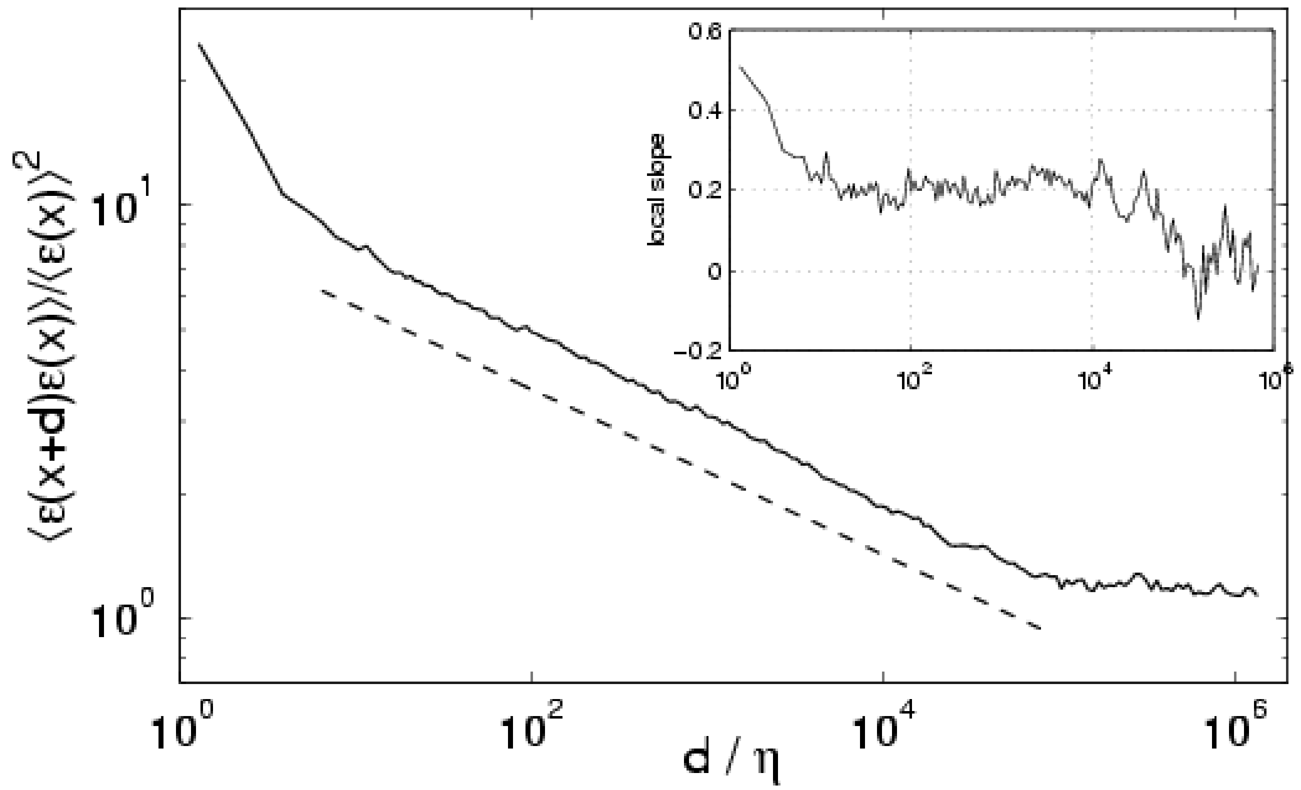
$$\left\langle \varepsilon_l^2 \right\rangle = \frac{1}{l^2}\int_l dx_1 \int_l dx_2 \left\langle \varepsilon(x_1)\varepsilon(x_2) \right\rangle$$



$\eta < l \leq L$

Europhys. Lett. 61 (2003) 756

Phys. Rev. E 71 (2005) 026309

Stochastic Energy Cascade Modelling

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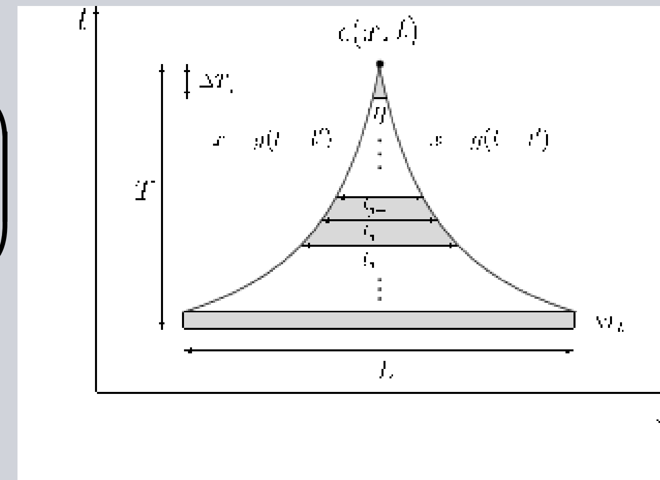
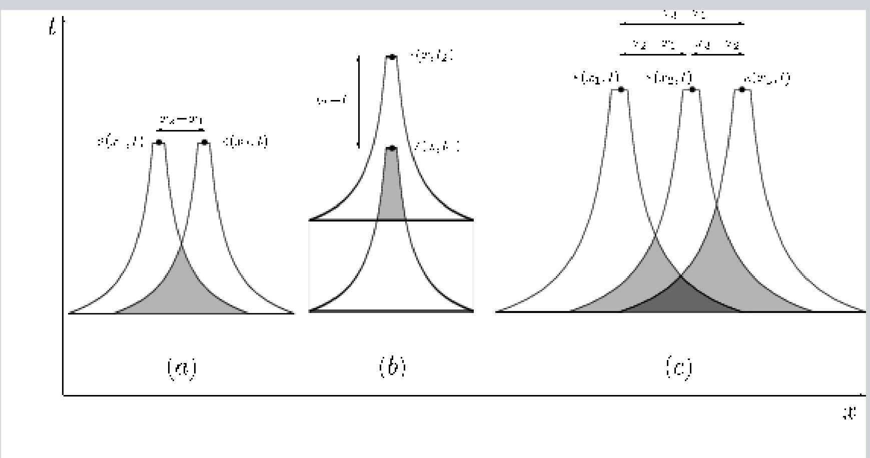
hierarchy of scales: $L \to L/\lambda \to \cdots \to L/\lambda^j \to \cdots \to L/\lambda^J = \eta$

random multiplication: $\varepsilon = q_1 \cdot q_2 \cdots q_j \cdots q_J = \exp\left(\sum_j \ln q_j\right)$

$$\varepsilon(x,t) = \exp\left(\int_{t-T}^{t} dt' \int_{x-L/2}^{x+L/2} dx' \, f(x-x', t-t') \, \gamma(x',t') \right)$$





$$\left\langle \varepsilon^{n_1}(x+d,t)\, \varepsilon^{n_2}(x,t) \right\rangle \propto \left(L/d\right)^{\tau(n_1,n_2)}$$

$$\tau(n_1,n_2) = \tau_{n_1+n_2} - \tau_{n_1} - \tau_{n_2}$$

$$\tau_n = \tau_2 (n - n^\alpha)/(2-2^\alpha)$$

Europhys. Lett. 61 (2003) 756

Test I: Three-Point Correlations

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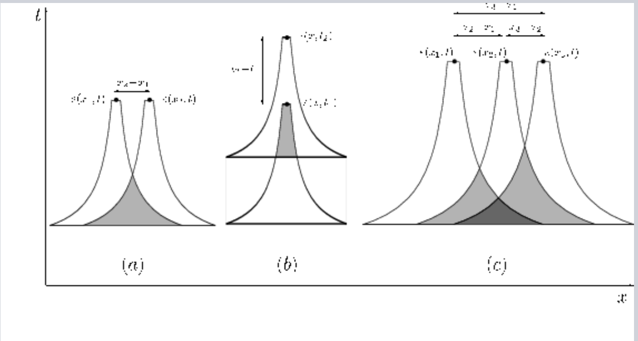
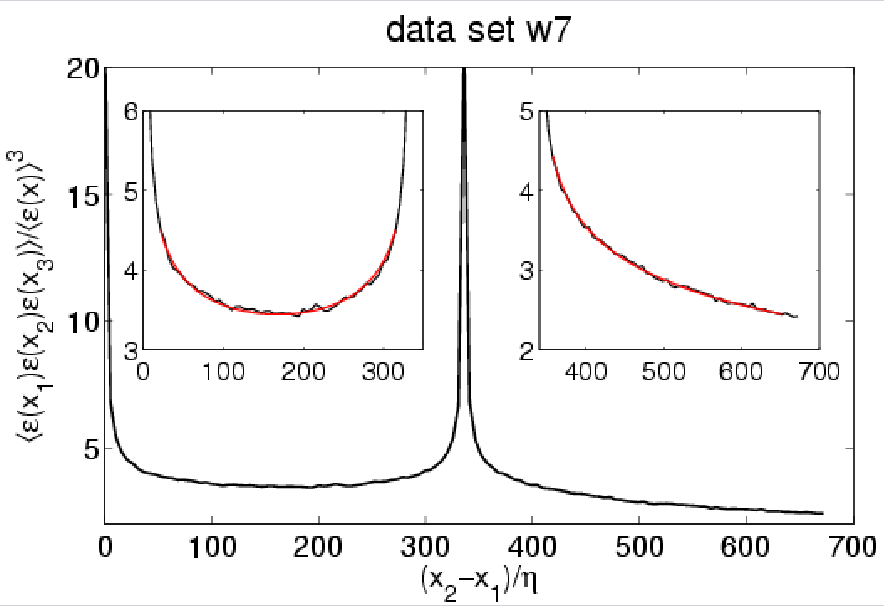
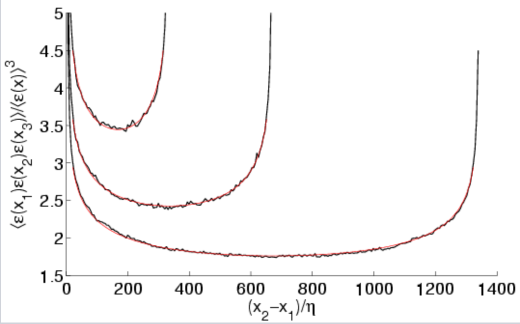
$$\left\langle \varepsilon^{n_1}(x_1,t)\,\varepsilon^{n_2}(x_2,t)\varepsilon^{n_3}(x_3,t)\right\rangle$$

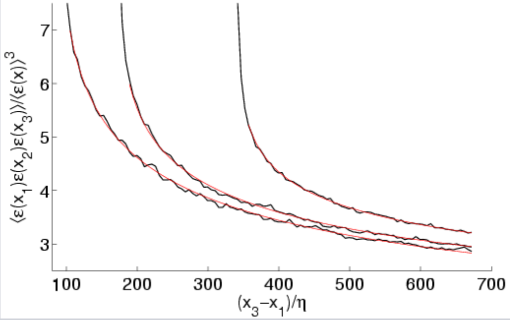
$$\propto \left(\frac{L}{x_3-x_1}\right)^{\alpha_{13}}\left(\frac{L}{x_2-x_1}\right)^{\alpha_{12}}\left(\frac{L}{x_3-x_2}\right)^{\alpha_{23}}$$

$$\alpha_{13}=\tau_{n_1+n_2+n_3}-\tau_{n_1+n_2}-\tau_{n_2+n_3}+\tau_{n_2} \qquad \alpha_{12}=\tau_{n_1+n_2}-\tau_{n_1}-\tau_{n_2} \qquad \alpha_{23}=\tau_{n_2+n_3}-\tau_{n_2}-\tau_{n_3}$$









wind tunnel,

R_λ= 860,

L/η = 1960

Phys. Lett. A 320 (2004) 247-253

Test II: Breakup Coefficients

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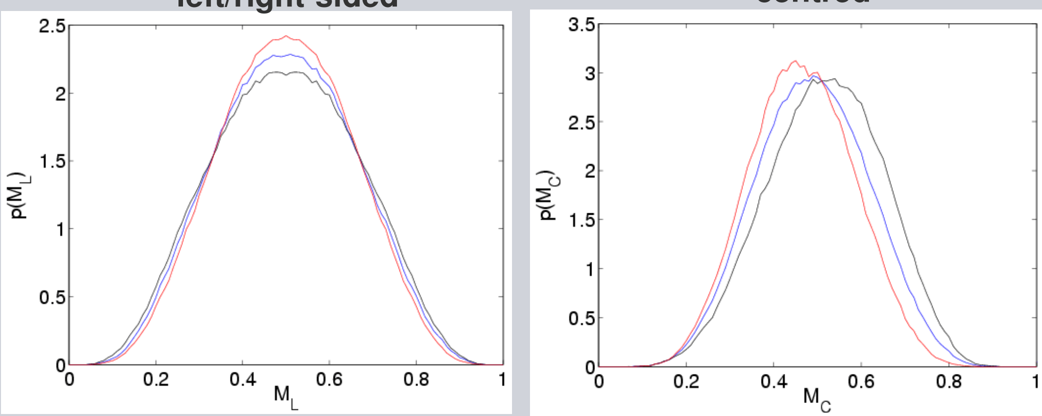
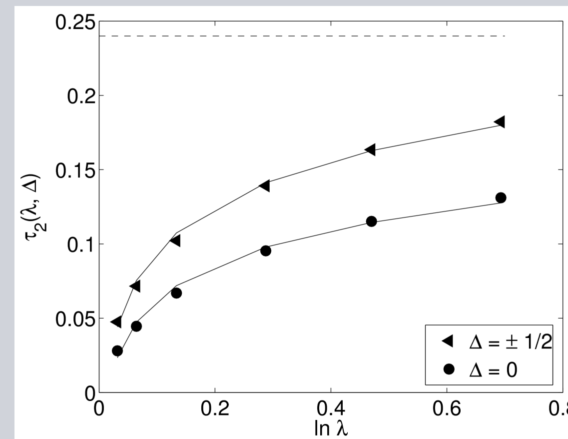
$$p(M_l \mid M_{\lambda l})$$

$$\lambda M \neq q$$

left/right-sided | centred





$$\lambda = 2, \quad \eta << l < L$$

scale correlations

$$M_l(\lambda) = \frac{\varepsilon_l}{\lambda\, \varepsilon_{\lambda l}}$$

$$\varepsilon_l = \frac{1}{l} \int_l \varepsilon(x)\, dx$$

Sreenivasan + Stolovitzky '95,
Molenaar, Herweijer + van de Water '95,
Pedrizzetti, Novikov + Praskovsky '96

Test III: Markov Picture

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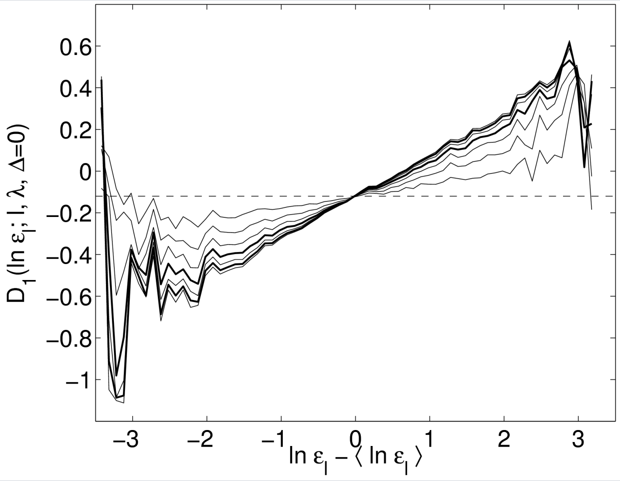
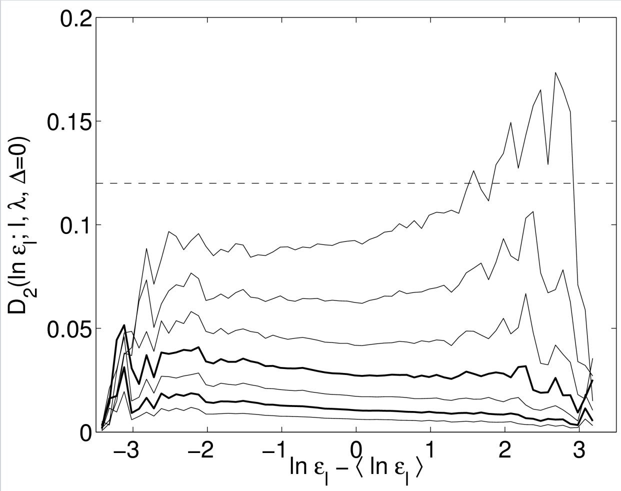


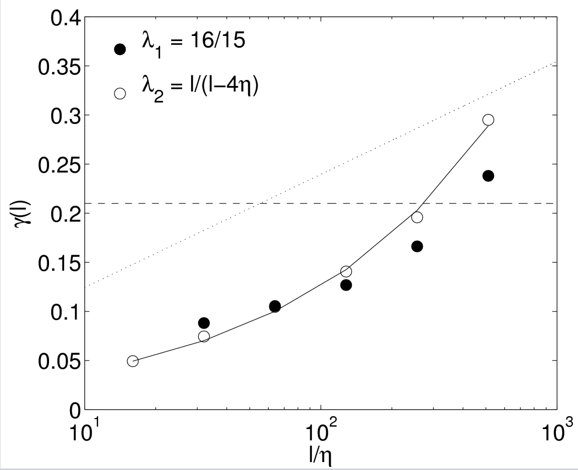
$$D_n(x,l) = \lim_{\lambda \to 1} \frac{1}{n! \ln\lambda} \left\langle \ln^n\left(\lambda M_l(\lambda)\right) \middle| x_l \right\rangle_c$$

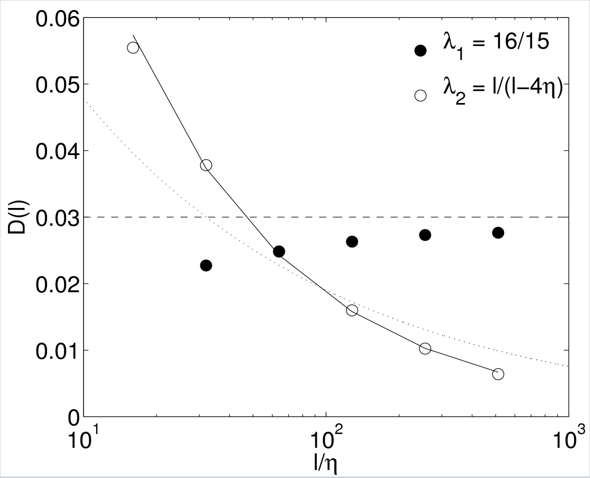
$$\partial_t \, p(x) = \sum_{n=1}^{\infty} (-1)^n \, \partial_x^n \, D_n(x) \, p(x)$$

$$x_t = \ln \varepsilon_l \qquad t = \ln(L/l)$$

Naert, Friedrich + Peinke '97
Marcq + Naert '98







again: scale correlations

Next Step: Modelling the Velocity Field

u

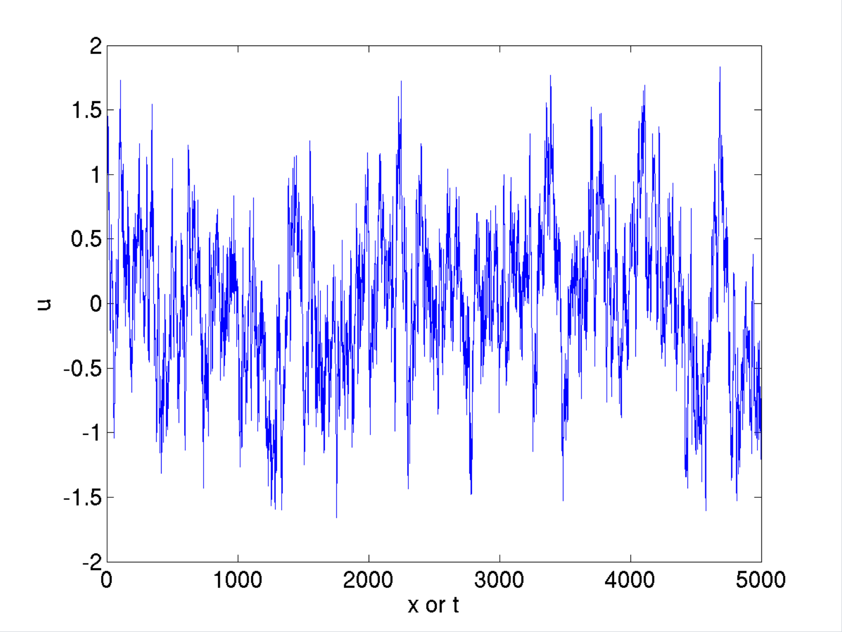
Standard (IEC 61400) Modelling of Turbulent Wind Fields

Jakob Mann model (Prob. Engng, Mech. **13** (1998), 269-282)

$$\boldsymbol{u}(\boldsymbol{x})=\int e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,d\boldsymbol{Z}(\boldsymbol{k})$$

$$\langle dZ_i^*(k)\,dZ_j(k)\rangle=\Phi_{ij}(k)\,dk_1\,dk_2\,dk_3$$

$$\Phi_{ij}(k)=\frac{E(k)}{4\pi k^4}(\delta_{ij}k^2-k_i k_j)$$



Standard vs. Real Wind Traces: Gaussian vs. Non-Gaussian

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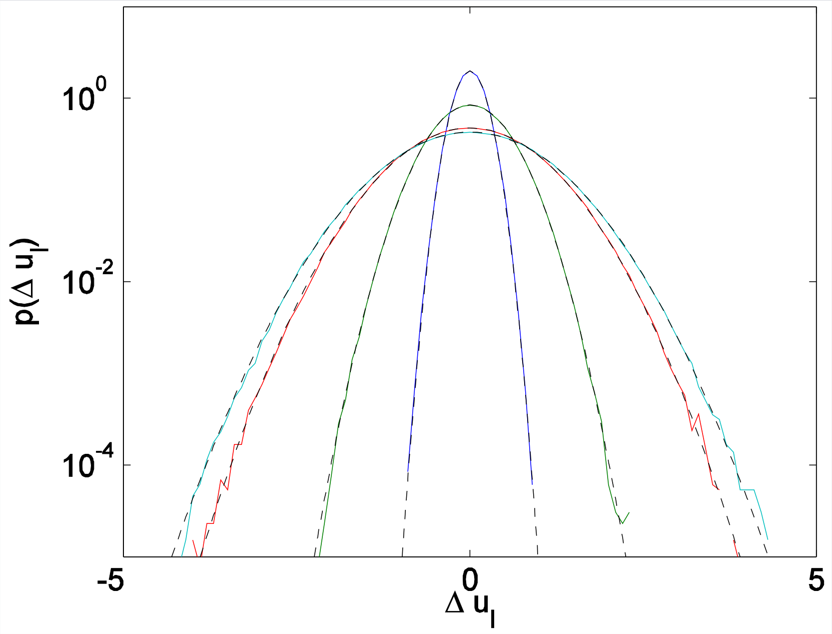
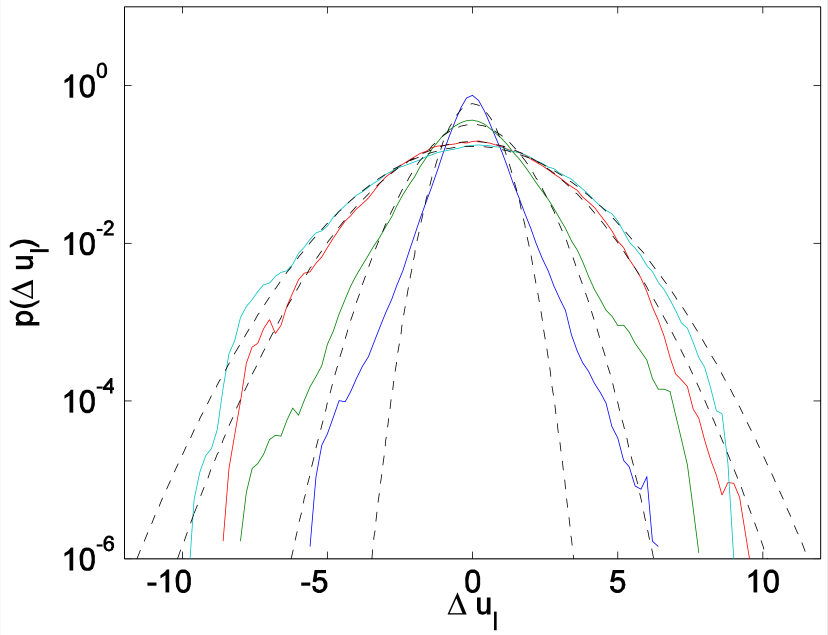
distributions of velocity increments $\Delta u_l(x) = u(x+l) - u(x)$

(standard normal turbulence model)



(observed atmospheric boundary layer)



$l = 1024m$ (turquoise), $32m$ (red), $8m$ (green), $1m$ (blue)

Standard vs. Real Wind Traces: Non-Intermittent vs. Intermittent

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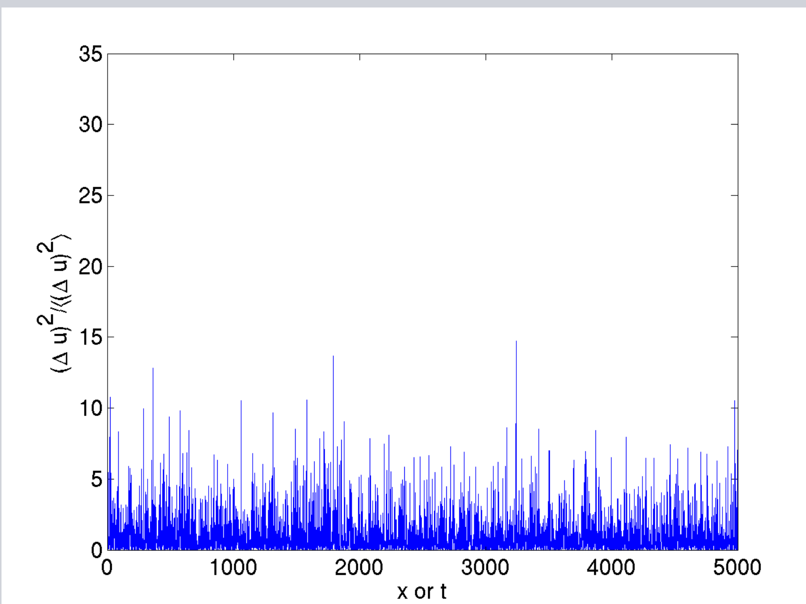
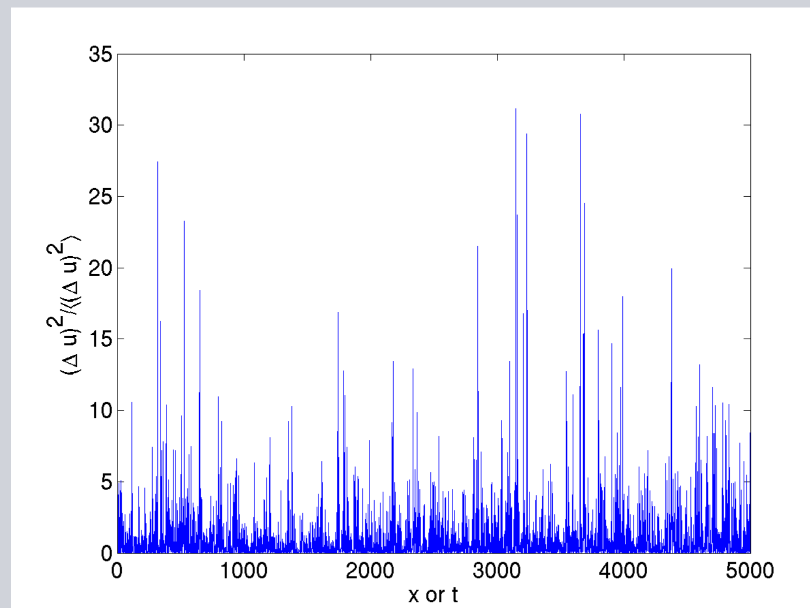
volatility of velocity increments $\Delta u(x)^2 = \left[u(x+\Delta x) - u(x)\right]^2$

(standard normal turbulence model)



$\Delta x = 1m$

(observed atmospheric boundary layer)



Multifractal Extension of Standard Normal Turbulence Modelling

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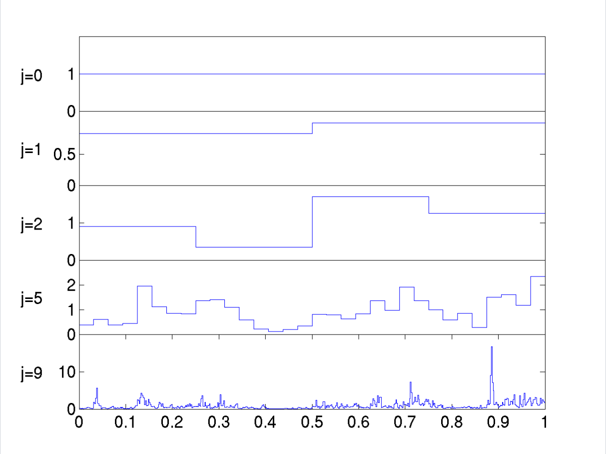
standard normal turbulence model

$$E(k) = c\,L^{5/3} / (1 + L^2 k^2)^{5/6}$$

$$u(x) = \sum_k \sqrt{E(k)}\, n_k\, e^{ikx}$$

$$\Delta u_{normal} = u(x + \Delta x) - u(x)$$

multifractal cascade process



$$\Delta u_{mf} = -\partial V(u_{mf}) + \sqrt{M(x)}\,\Delta u_{normal}(x)$$

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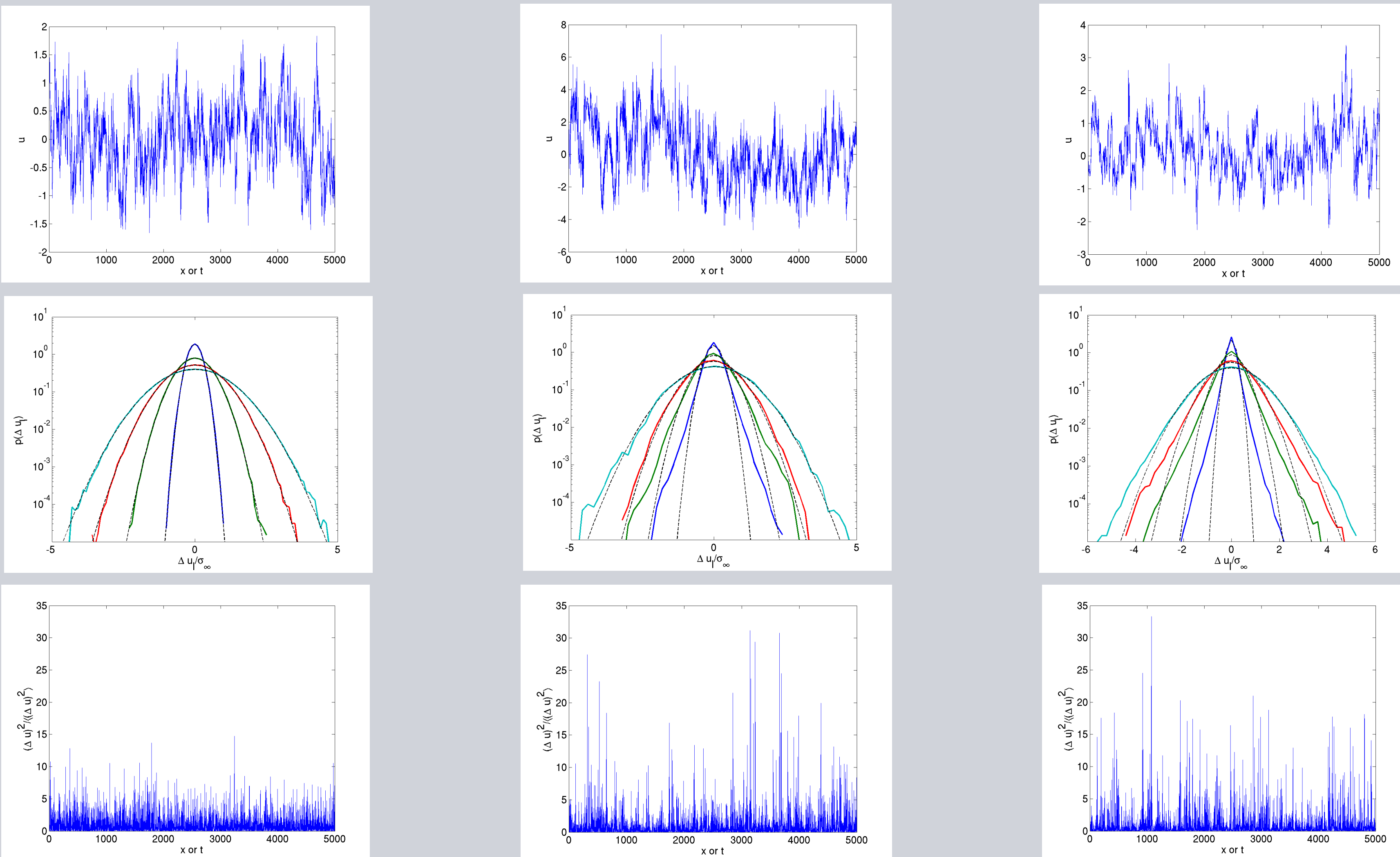
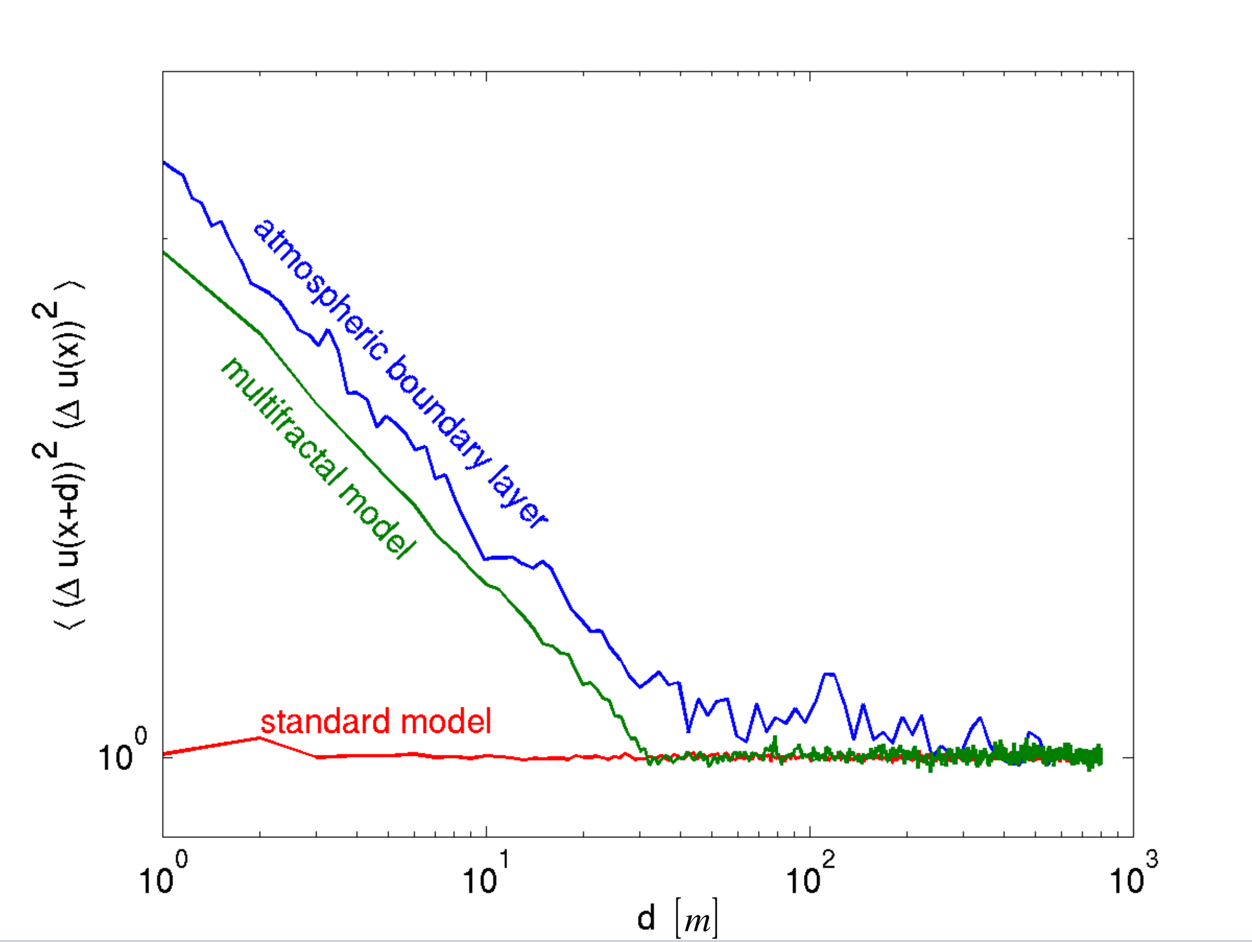
Wind Traces: Standard - Real - Multifractal



Wind Gust Clustering: Standard - Real - Multifractal



correlation length ≈ 30m,

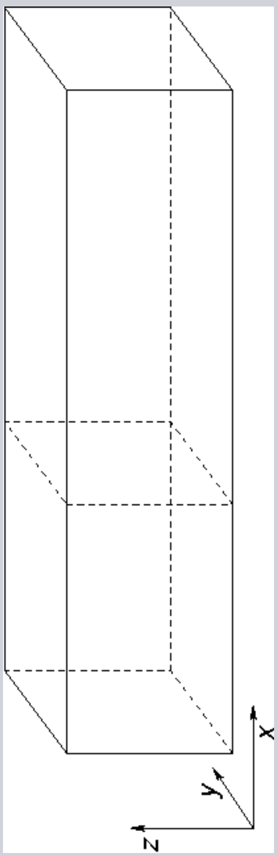
correlation time ≈ 5sec

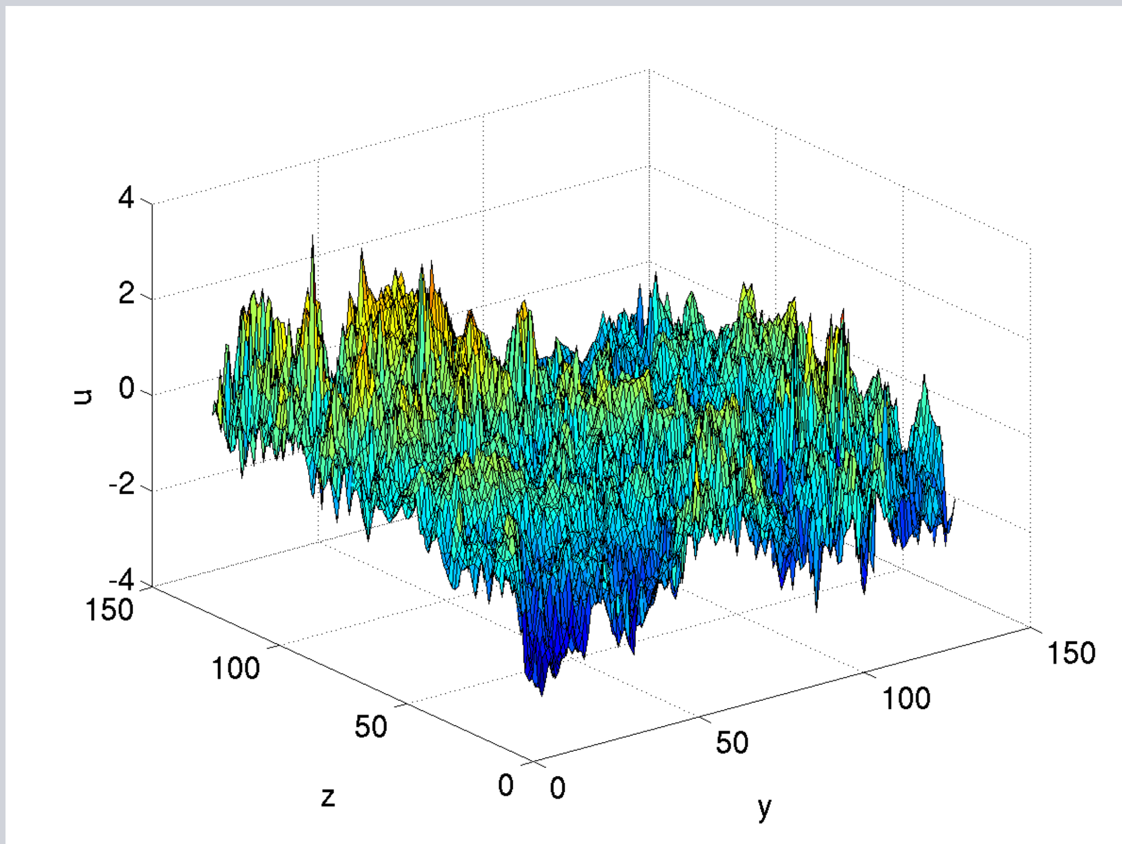
length of rotor blade ≈ 40m,

resonance frequency ≈ 0.5Hz

More Multifractal Modelling: from Wind Traces to Wind Fields

$$u(x, y, z)$$





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Conclusion & Outlook

- Cascade captures properties of turbulence well
- Realistic turbulence modelling of wind fields
- Developed method is very general

- Future efforts
  - ➔ on-site wind-data calibration
  - ➔ extreme value statistics
  - ➔ wind-turbine interaction modelling
  - ➔ meander modelling

